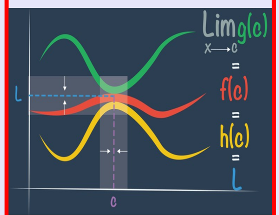
Calculus I

Final Exam

Review

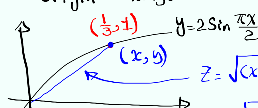


Feb 19-8:47 AM

An object moves along $y = 2\sin\frac{\pi x}{2}$.

As it moves at the point $(\frac{1}{3}, 1)$, x-coordinate increases at $\sqrt{10}$ cm/s. $\frac{dx}{dt} = \sqrt{10}$ cm/s

How fast is the distance of the object and the origin changes at that time.



$Z = \sqrt{(x-0)^2 + (y-0)^2}$

$Z = \sqrt{x^2 + y^2}$

$Z = \sqrt{(\frac{1}{3})^2 + 1^2}$

$Z = \sqrt{\frac{10}{9}} = \frac{\sqrt{10}}{3}$

$Z^2 = x^2 + y^2$

$Z^2 = x^2 + \left(2\sin\frac{\pi x}{2}\right)^2 \rightarrow Z^2 = x^2 + 4\sin^2\frac{\pi x}{2}$

we want $\frac{dz}{dt}$ at $(\frac{1}{3}, 1)$

$\sin 2A = 2\sin A \cos A$

$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 4 \cdot \boxed{2\sin\frac{\pi x}{2} \cdot \cos\frac{\pi x}{2}} \cdot \frac{\pi}{2}\frac{dx}{dt}$

$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + \cancel{4}^{2} \cdot \boxed{\sin \pi x} \cdot \frac{\pi}{\cancel{2}}\frac{dx}{dt}$

$z\frac{dz}{dt} = x\frac{dx}{dt} + \pi \sin \pi x \frac{dx}{dt}$

$\frac{\sqrt{10}}{3}\frac{dz}{dt} = \frac{1}{3} \cdot \sqrt{10} + \pi \sin\frac{\pi}{3} \cdot \sqrt{10}$

$\frac{1}{3}\frac{dz}{dt} = \frac{1}{3} + \pi \cdot \frac{\sqrt{3}}{2} \rightarrow \frac{dz}{dt} = \frac{\frac{1}{3} + \frac{\pi\sqrt{3}}{2}}{\frac{1}{3}}$

$\frac{dz}{dt} = 1 + \frac{3\pi\sqrt{3}}{2}$ cm/s.

Dec 10-7:27 AM

Find two numbers whose product is 100 and whose sum is minimum.

x & y

$xy = 100$

$y = \frac{100}{x}$

Minimized

$x + y$

$x + \frac{100}{x}$

$f(x) = x + \frac{100}{x}$

$f'(x) = 1 - \frac{100}{x^2}$

$f''(x) = \frac{200}{x^3}$

$f'(x) = 0$

$1 - \frac{100}{x^2} = 0$

$x = \pm 10$

$\boxed{x = 10} \rightarrow f''(10) > 0$

$x = 10$

$y = \frac{100}{10} = 10$

$x = -10 \rightarrow f''(-10) < 0$

Min.

Max

10 & 10

Dec 10-7:40 AM

Find f_{ave} for $f(x) = \sin x \cos^4 x$ on $[0, \pi]$.

QI & QII

$\sin x \geq 0$

$\cos^4 x \geq 0$

$f(x) \geq 0$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{\pi - 0}\int_0^\pi \sin x \cos^4 x\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$x = 0 \rightarrow u = 1$

$x = \pi \rightarrow u = -1$

$$= \frac{1}{\pi}\int_1^{-1} u^4 \cdot -du$$

$$= \frac{1}{\pi}\int_{-1}^{1} u^4\,du$$

$$= \frac{1}{\pi} \cdot \frac{u^5}{5}\Big|_{-1}^{1}$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$= \frac{1}{5\pi}\left[1^5 - (-1)^5\right]$$

$$= \boxed{\frac{2}{5\pi}}$$

Dec 10-7:46 AM

Use $\underbrace{\text{linear approximation}}$ to estimate $\sqrt[3]{1001}$.

$\sqrt[3]{1001} \approx \sqrt[3]{1000} = 10$

$f(x) = \sqrt[3]{x}$ $\qquad$ $f(x) \approx f(a) + f'(a)(x-a)$

$a = 1000$ $\qquad$ $\sqrt[3]{x} \approx \sqrt[3]{1000} + f'(1000)(x-1000)$

$\sqrt[3]{x} \approx 10 + \frac{1}{300}(x-1000)$

$f(x) = x^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$ $\qquad$ $f'(1000) = \frac{1}{3\sqrt[3]{1000^2}} = \frac{1}{300}$

$\sqrt[3]{1001} \approx 10 + \frac{1}{300}(10001-1000)$

$\sqrt[3]{1001} \approx 10.00333$

$\frac{3001}{300} \approx 10.0033333\bar{3}\ldots$ $\qquad$ $\approx 10 + \frac{1}{300} = \boxed{\frac{3001}{300}}$

Dec 10-7:55 AM

$f(x) = \frac{8(\sqrt{x}-1)}{x}$, $f'(x) = \frac{4(2-\sqrt{x})}{x^2}$, $f''(x) = \frac{2(\sqrt{x}-3)}{x^3}$

$\underbrace{x \neq 0,\ x \geq 0}$

Domain $(0,\infty)$

No Y-Int

x-Int $\to$ $\sqrt{x}-1=0$

$(1,0)$ $\qquad$ $x=1$

C.P. $f'(x)=0$ or und.

$2-\sqrt{x}=0$

$x=4$

$(4,2)$

P.I.P.

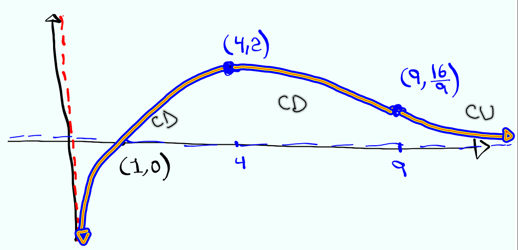
$\sqrt{x}-3=0$

$x=9$

$(9, \frac{16}{9})$

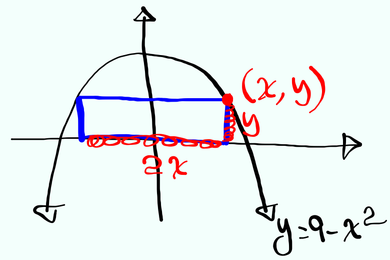
x	0	4	9	∞
$f'(x)$	+	• −	−	
$f''(x)$	−	−	• +	
$f(x)$	↗	↘	↘	

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{8(\sqrt{x}-1)}{x} = 0$ H.A. $\qquad$ VA $x=0$



Dec 10-8:03 AM

Dimension of rectangle with largest area



Area $= 2xy$

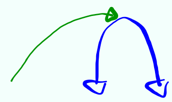
$= 2x(9-x^2)$

$f(x) = 18x - 2x^3$

$f'(x) = 18 - 6x^2$

$f''(x) = -12x$

$x>0 \quad f''<0$



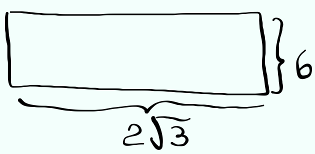
Max

$f'(x)=0$

$18 - 6x^2 = 0$

$3 - x^2 = 0$

$x = \sqrt{3}$



Dec 10-8:16 AM

abs. Max & abs. Min

$f(x) = x^3 - 6x^2 + 5$

on $[0,5]$

Polynomial

Cont. & Diff.

$(-\infty, \infty)$

$f(0) = 5$

$f(5) = -20$

$f'(x) = 3x^2 - 12x$

$f'(x) = 0 \qquad 3x(x-4) = 0$

$x=0 \qquad x=4$

$f(4) = -27$

Abs. Max 5 at $x=0$

" Min -27 at $x=4$

Dec 10-8:21 AM

Abs. Max & abs. Min of $f(x) = \frac{1}{x}$

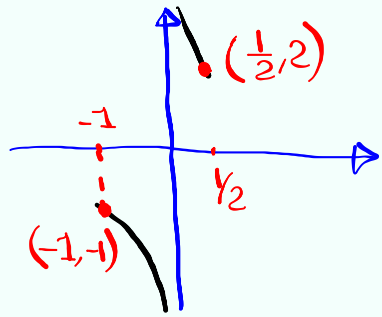
on $[-1, \frac{1}{2}]$

$f(-1) = -1$

$f(\frac{1}{2}) = 2$

$f'(x) = \frac{-1}{x^2}$

$f'(x) \neq 0$



Dec 10-8:25 AM